EE362: Signals and Systems Name:----------

Spring 2019

Homework #10 (Due: Tuesday, April 16, 2019)

- Circle or box your answer (don't forget units).
- **Show** your steps.
- Use 8.5" x 11" paper, and trim off any ragged edges.
- **Staple** your work together and write your name neatly.

1. Consider the signal whose Fourier transform is shown in Figure 1. Let $x_s(t) = x(t)p(t)$ represent the sampled signal. Draw $|X_s(\omega)|$ for the following cases: (a) $T = \pi/15$ and (b) $T = 2\pi/15$. [6 pts]

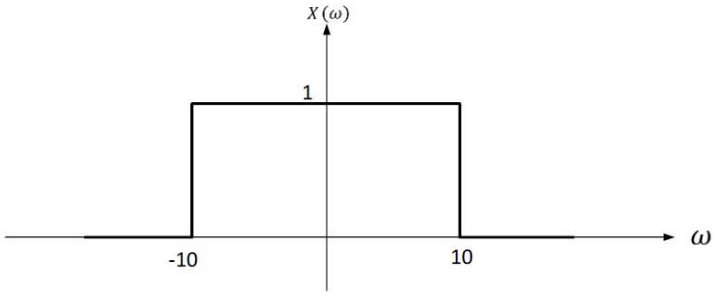


Figure 1

2. Repeat problem 1 for the signal whose transform is shown in Figure 2. [6 pts]

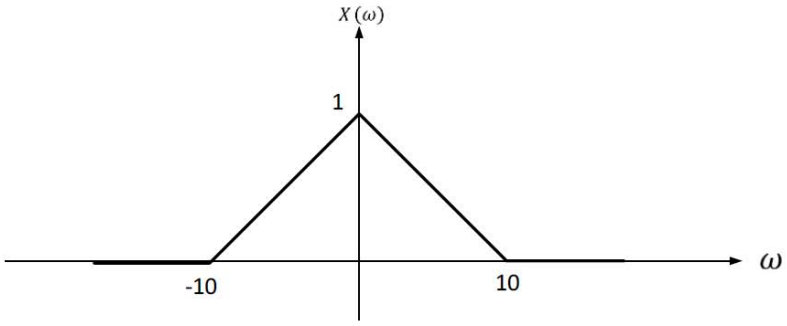


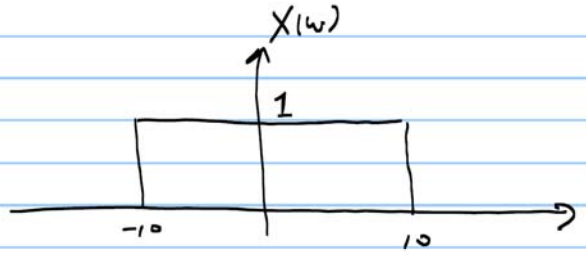
Figure 2

3. Work out the following problems from your textbook (Edward W. Kamen and Bonnie S. Heck, Fundamentals of Signals and Systems Using the Web and MATLAB, 3rd Ed, Prentice Hall, 2006)

1. Problem 5.39 parts (b)-(i) and (b)-(ii). [5 points]

2. Problem 5.40 parts (a), (c), and (d). [5 points]

(1)

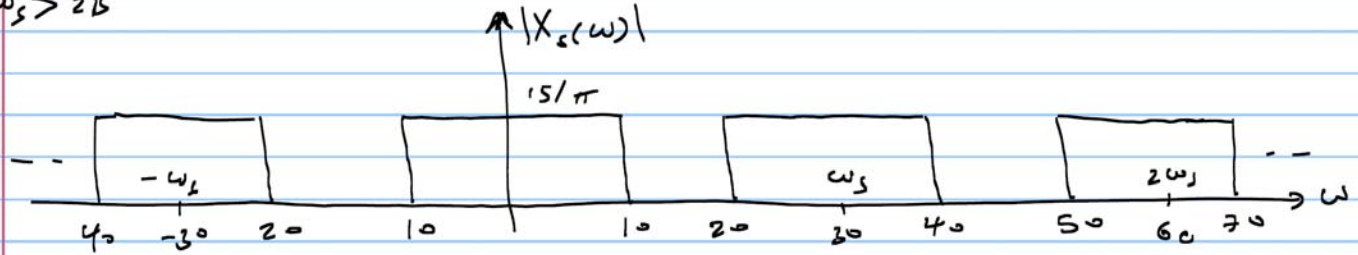


$\Rightarrow A = 1$

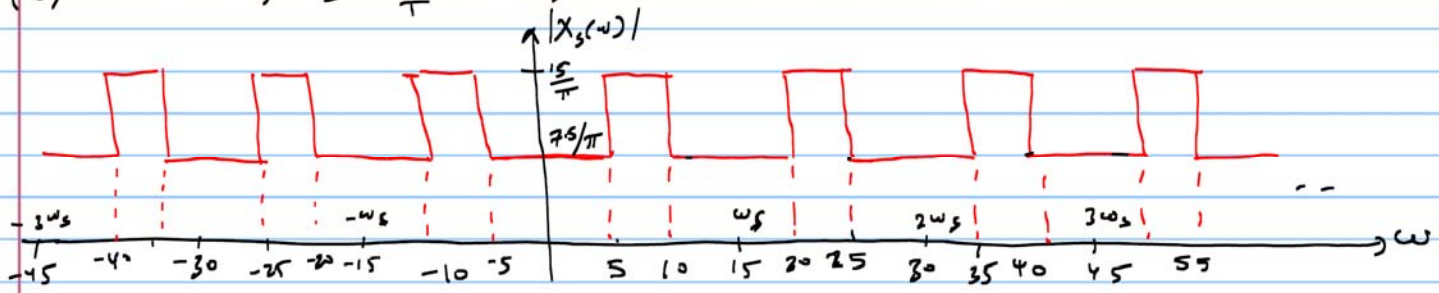
$B = 10$

(a) $T = \pi/15$; $\omega_s = \frac{2\pi}{T} = 30$; $A/T = 15/\pi$

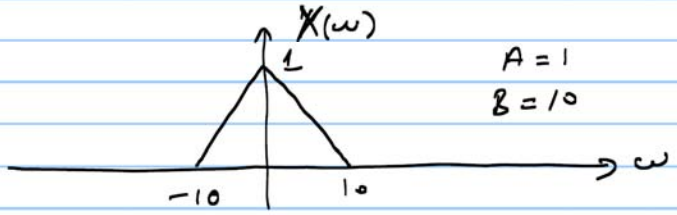
$\therefore \omega_s > 2B$



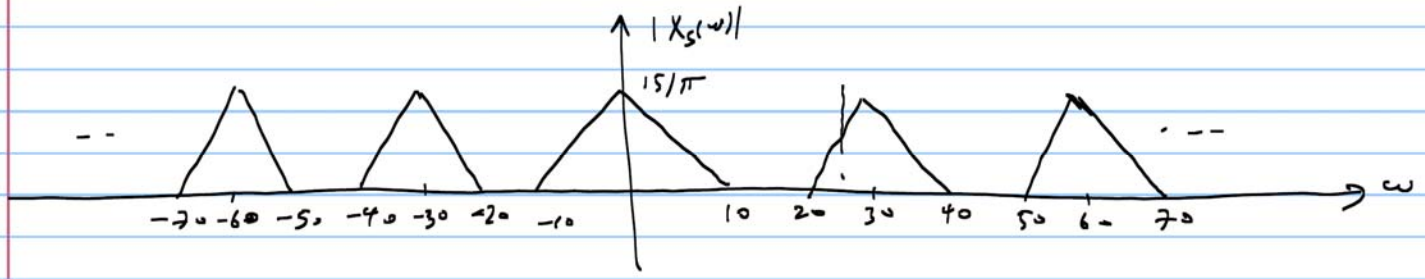
(b) $T = 2\pi/15$; $\omega_s = \frac{2\pi}{T} = 15$; $A/T = 7.5/\pi$; $\omega_s < 2B$



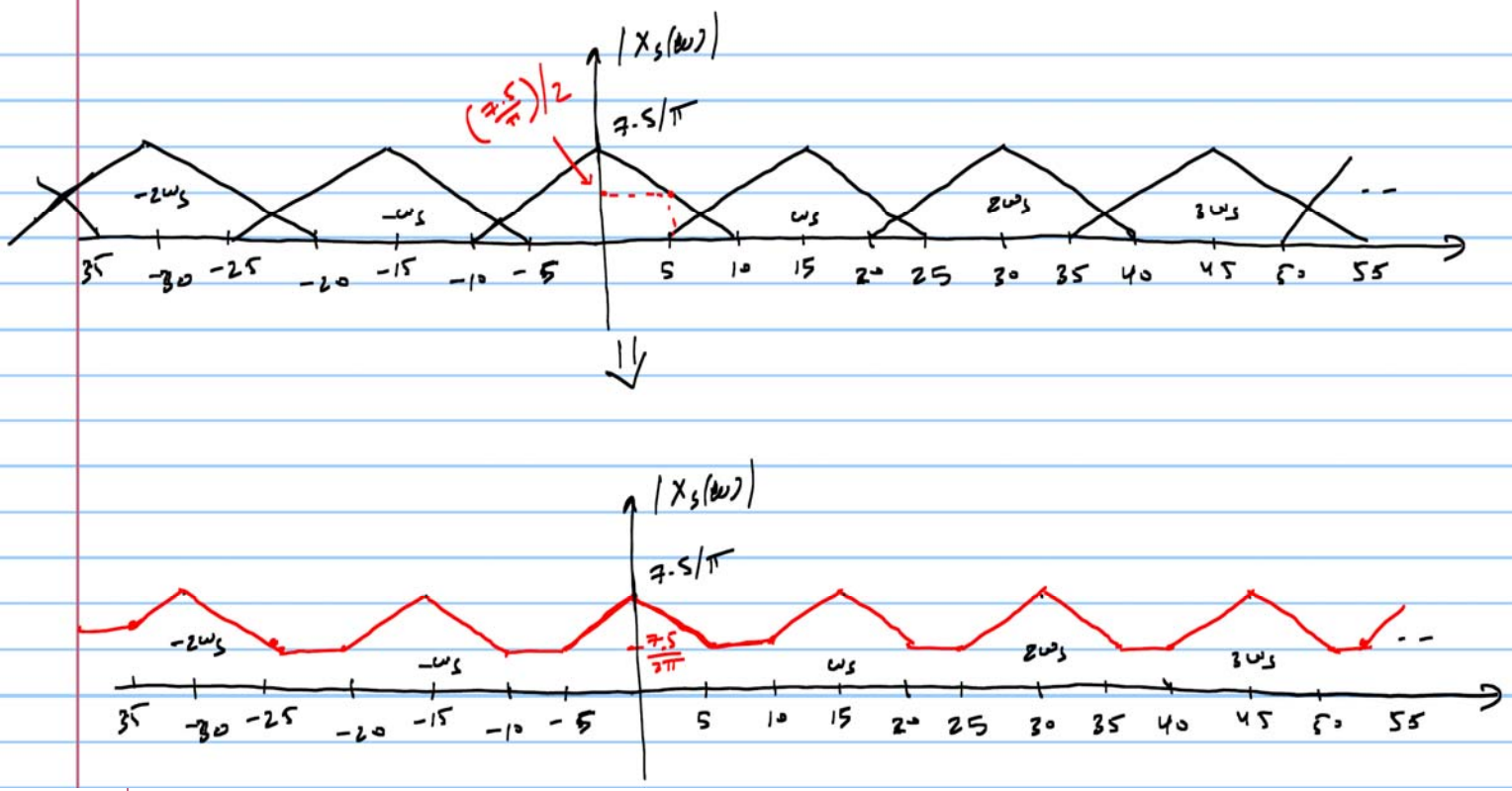
(2)



(a) $T = \pi/15 \Rightarrow \omega_s = 30 > 2B$; $A/T = 15/\pi$

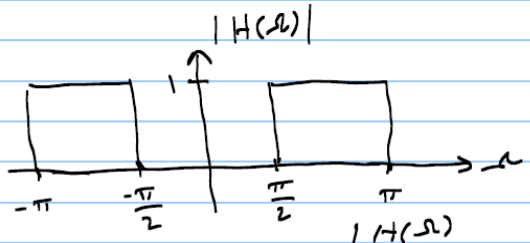


(b) $T = 2\pi/15 \Rightarrow \omega_s = 15 < 2B$; $A/T = 7.5/\pi$


$|X_s(\omega)|$
$7.5/\pi$
$(\frac{7.5}{\pi})/2$
$-2\omega_s$
$-\omega_s$
ω_s
$2\omega_s$
$3\omega_s$
-35 -30 -25 -20 -15 -10 -5 5 10 15 20 25 30 35 40 45 50 55
$|X_s(\omega)|$
$7.5/\pi$
$\frac{7.5}{2\pi}$
$-2\omega_s$
$-\omega_s$
ω_s
$2\omega_s$
$3\omega_s$
-35 -30 -25 -20 -15 -10 -5 5 10 15 20 25 30 35 40 45 50 55

(3)

1. $H(\Omega) = \begin{cases} e^{-j3\Omega} & , \frac{\pi}{2} \le |\Omega| \le \pi \\ 0 & , 0 \le |\Omega| < \frac{\pi}{2} \end{cases}$



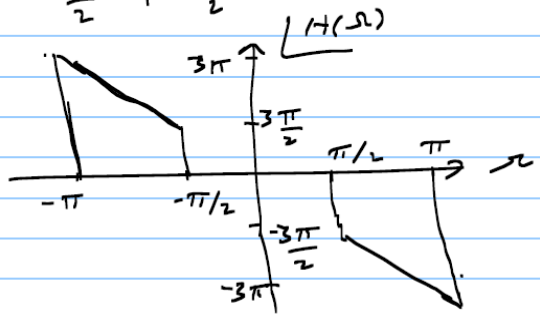
(b)

(i) $x[n] = \cos(\pi n/4)$, $n = 0, \pm 1, \pm 2, \cdots$

$y[n] = |H(\frac{\pi}{4})| \cos\left(\frac{\pi n}{4} + \angle H(\frac{\pi}{4})\right)$

$H(\frac{\pi}{4}) = 0$

$\therefore y[n] = 0$



(ii) $x[n] = \cos(\frac{3\pi}{4} n)$

$H(3\pi/4) = e^{-j\frac{9\pi}{4}} \Rightarrow |H(\frac{3\pi}{4})| = 1 \;\&\; \angle H(3\pi/4) = -\frac{9\pi}{4}$

$\therefore y[n] = \cos\left(\frac{3\pi}{4} n - \frac{9\pi}{4}\right) = \cos\left(\frac{3\pi}{4} n - \frac{\pi}{4}\right)$

2. (a) $H(\Omega) = P_\pi(\Omega) + \left(1 - \frac{2|\Omega|}{\pi}\right) P_\pi(\Omega)$, for $-\pi \le \Omega \le \pi$

$\therefore h[n] = \frac{1}{2}\,\text{sinc}\left(\frac{n}{2}\right) + \frac{1}{4}\,\text{sinc}^2\left(\frac{n}{4}\right)$, $n = 0, \pm 1, \pm 2 \cdots$

(c) $y[n] = 2H(0) + H\left(\frac{\pi}{4}\right)\sin\left(\frac{\pi n}{4}\right) + H\left(\frac{\pi}{2}\right)2\sin\left(\frac{\pi n}{2}\right)$

$\therefore y[n] = 4 + 1.5\sin\left(\frac{\pi n}{4}\right) + 2\sin\left(\frac{\pi n}{2}\right)$

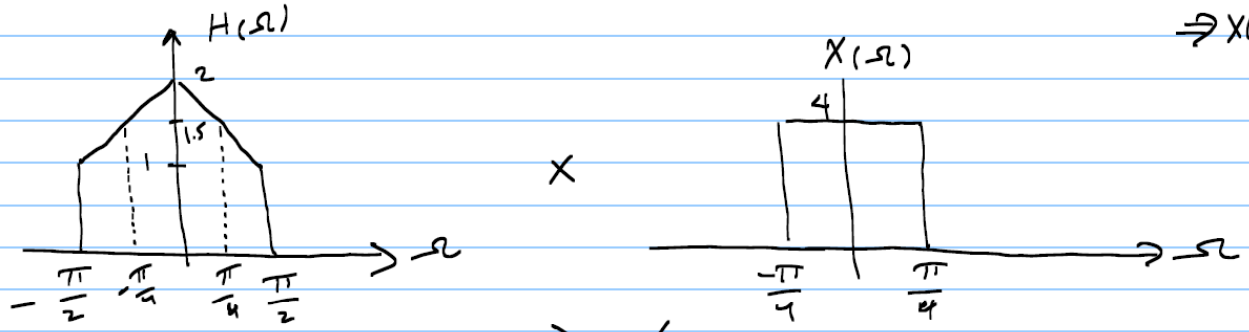
(d) $X(\Omega) = 4P_\pi(2\Omega)$ for $-\pi \le \Omega \le \pi$

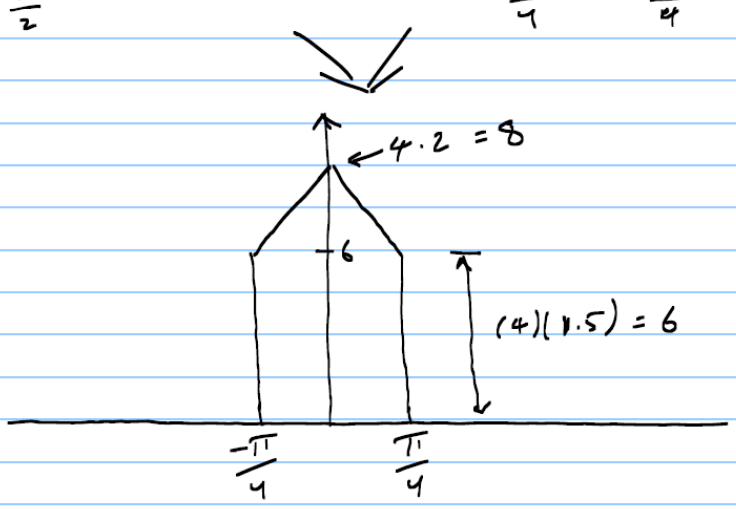
$Y(\Omega) = 4P_\pi(2\Omega)\left(P_\pi(\Omega) + \left(1 - \frac{2|\Omega|}{\pi}\right)P_\pi(\Omega)\right) = ?$

$P_\tau(\Omega) = \begin{cases} 1, & \frac{\tau}{2} \le \Omega \le \tau/2 \\ 0, & \text{o.w.} \end{cases}$

$\therefore P_\pi(2\Omega) = \begin{cases} 1, & -\frac{\pi}{2} \le 2\Omega \le \pi/2 \\ 0, & \text{o.w.} \end{cases} = \begin{cases} 1, & -\frac{\pi}{4} \le \Omega \le \frac{\pi}{4} \\ 0, & \text{o.w.} \end{cases} = P_{\frac{\pi}{2}}(\Omega)$

$\Rightarrow X(\Omega) = 4P_{\frac{\pi}{2}}(\Omega)$





$$\therefore Y(\Omega) = 6\,P_{\frac{\pi}{2}}(\Omega) + 2\left(1 - \frac{2|\Omega|}{\pi/2}\right) P_{\frac{\pi}{2}}(\Omega) \quad ; \; \tau = \pi/2$$

$$\therefore y[n] = 6\left(\frac{\tau}{2\pi}\right) \operatorname{sinc}\left(\frac{\tau n}{2\pi}\right) + 2\left(\frac{\tau}{4\pi}\right) \operatorname{sinc}^2\left(\frac{\tau n}{4\pi}\right)$$

$$= \frac{3}{2} \operatorname{sinc}\left(\frac{n}{4}\right) + \frac{1}{4} \operatorname{sinc}^2\left(\frac{n}{8}\right), \quad n = 0, \pm 1, \pm 2, \cdots$$